

# Design of Elastic Band Structures with Broken Symmetry via Spatio-Temporal Modulations of Elasticity

**Pierre A. Deymier<sup>1</sup>, Keith Runge<sup>1</sup>, Vitthal Gole<sup>1</sup>, Pierre Lucas<sup>1</sup>, Jerome O. Vasseur<sup>2</sup> and Nicholas Boechler<sup>3</sup>**

<sup>1</sup> Department of Materials Science and Engineering, University of Arizona, Tucson, AZ 85721 USA  
[deymier@email.arizona.edu](mailto:deymier@email.arizona.edu), [krunge@email.arizona.edu](mailto:krunge@email.arizona.edu), [vithalgole@gmail.com](mailto:vithalgole@gmail.com), [pierre@email.arizona.edu](mailto:pierre@email.arizona.edu)

<sup>2</sup> Institut d'Electronique, de Micro-électronique et de Nanotechnologie, UMR CNRS 8520, Cité Scientifique, 59652 Villeneuve d'Ascq Cedex, France  
[Jerome.vasseur@univ-lille1.fr](mailto:Jerome.vasseur@univ-lille1.fr)

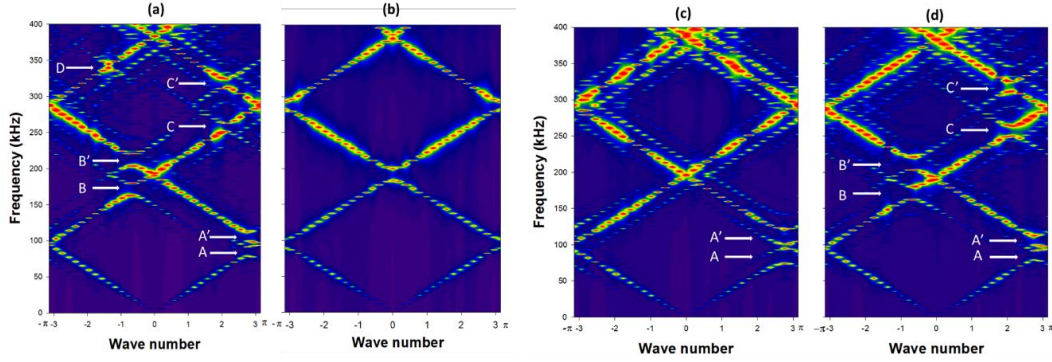
<sup>3</sup> Department of Mechanical Engineering, University of Washington, Seattle, WA 98195 USA  
[boechler@uw.edu](mailto:boechler@uw.edu)

**Abstract:** Periodic spatio-temporal modulations (STM) of the elastic properties of materials are used to break time and parity symmetry of elastic waves. The shape of the STM is shown to affect band structure asymmetry, independent of its period.

Several approaches offer pathways to access the deliberate design of elastic media with broken symmetry, including intrinsic and extrinsic methods. Symmetry in intrinsic systems is broken without addition of energy from the outside. In contrast, symmetry can be broken extrinsically by moving fluids<sup>1,2</sup>, gyroscopic inclusions<sup>3</sup>, or directed and externally driven spatio-temporal modulations of the properties of the medium<sup>4,5</sup>.

In one-dimension, a periodic spatio-temporal modulation can effectively create a moving superlattice of the elastic constants in the medium. The time-dependent superlattice breaks time-reversal and parity symmetry leading to bulk phonon modes with non-conventional topology. These bulk topological elastic states do not possess the conventional mirror symmetry in momentum space leading to non-reciprocity in the direction of propagation of the waves. The spatio-temporal modulation results in non-linear effects that produce features reminiscent of Brillouin scattering and hybridization band gaps. The hybridization gaps result from interactions between the Bloch modes of the medium in absence of the temporal variation of the modulation and Brillouin scattering-like harmonic modes. Here, we also show that the shape of the periodic spatially-varying modulation does matters. We consider various shapes of modulations with the same period and velocity. First we consider a general periodic superlattice that takes the form of a periodic superposition of Gaussian functions. Two additional superlattices represent Fourier expansions (in terms of sinusoidal functions) of the general superlattice at different orders of truncation. The lowest order of truncation actually corresponds to a single sinusoidal modulation. The next order in truncation includes multiple sinusoidal terms. Note that all Fourier components possess the same velocity. We calculate the band structures associated with the three modulations using Spectral Energy Density method<sup>6</sup>.

Figure 1(b) is characteristic of the band structure of a superlattice that does not evolve in time. It consists of the usual folded bands with gaps opening at the edges of the Brillouin zone and at the wave number origin. In contrast, the time-dependent superlattice with the general spatial modulation possesses a band structure (Fig. 1(a)) that exhibits a number of features characteristic of broken symmetries. These features take the form of hybridization gaps. The interaction between elastic waves with frequency  $f_0$  and a spatiotemporal modulation of the elastic constants leads to a frequency splitting that resembles Brillouin scattering. The frequency of the scattered modes contains harmonics of the frequency associated with the moving modulation:  $f_n = f_0 \pm nF$ , where  $F = V/L$  and  $V$  is the velocity of the modulation and  $L$  is its period. These scattered modes appear as faint bands parallel to the folded bands of the static superlattice. The scattered modes hybridize with the static folded bands to form band gaps. The gaps form asymmetrically with respect to the wave number origin. For instance, the gaps A and A' result from the hybridization between a first order harmonic ( $n=1$ ) and the first and second bands of the static system.



**Figure 1** Band structure of (a) moving general modulation composed of a superposition of Gaussian functions, (b) same general modulation but static (velocity is zero). (c) Moving modulation including a single sinusoidal function representing the Fourier series of the general modulation to first-order and (d) moving modulation including multiple sinusoidal functions representing the Fourier series of the general modulation to third order. The band structures are calculated using the SED method. To enhance the contrast, the color contour plots represent the logarithm of the SED intensity.

These gaps occur in the positive wave number side of the Brillouin zone without equivalent on the negative side. Hybridization between second harmonics ( $n=2$ ) and the second and third static bands produces the gaps labeled B and B'. These gaps form only on the negative side of the Brillouin zone. C and C' are asymmetric hybridization gaps between third harmonic ( $n=3$ ) and the third and fourth folded bands. Finally, hybridization of the fourth folded band and  $n=4$  harmonics forms gap D on the negative side of the Brillouin zone. Asymmetric gaps of the type observed here are known to lead to unconventional wave topologies. The asymmetric band structure is characteristic of systems with broken parity and time-reversal symmetry. At frequencies within the gaps, bulk modes can only propagate in one direction, that is, they possess non-reciprocity in their direction of propagation. Therefore, these modes will also possess immunity to back scattering. In Figure 1(c), the band structure of the system with a single sinusoidal spatio-temporal modulation only shows the A and A' gaps. Figure 1(d), shows that when one approaches the Gaussian periodic modulation with a Fourier-like series of sinusoidal functions at several orders, one recovers the features A, A' and B, B', and C, C' but not the gap D.

We show with a simple theoretical model that shaping spatio-temporal stiffness modulations can be employed as a tool for elastic band structure design via the tailoring of specific symmetry breaking band features. Using spatio-temporal modulations of various forms offers a vast array of pathways to access new topological classes of materials and the controlled transport and reciprocity of electron, photon, and phonon trajectories in materials.

## References

- <sup>1</sup> R. Fleury, D.L. Sounas, C.F. Sieck, M.R. Haberman and A. Alu, *Science* 343, 516 (2014).
- <sup>2</sup> Q. Wang, Y. Yang, X. Ni, Y-L. Xu, X-C. Sun, Z-G. Chen, L. Feng, X-P. Liu, M-H. Lu and Y-F. Chen, *Scientific Reports* 5, 10880 (2015).
- <sup>3</sup> P. Wang, L. Lu, and K. Bertoldi, *Phys. Rev. Lett.* 115, 104302 (2015).
- <sup>4</sup> N. Swintek, S. Matsuo, K. Runge, J.O. Vasseur, P. Lucas and P.A. Deymier, *J. Appl. Phys.* 118, 063103 (2015).
- <sup>5</sup> C. Croenne, J. O. Vasseur, O. Bou Matar, M.-F. Ponge, P. A. Deymier, A.-C. Hladky-Hennion, and B. Dubus, *Appl. Phys. Lett.* in press.
- <sup>6</sup> J.A. Thomas, J.E. Turney, R.M. Iutzi, C.H. Amon, and A.J.H. McGaughey, *Phys. Rev. B* 81, 081411 (2010).